Generalized heat equations in supercritical function spaces

Franka Baaske *

Friedrich-Schiller-University of Jena, Germany

We deal with a generalized heat equation

$$\frac{\partial}{\partial t}u(x,t) + (-\Delta_x)^\alpha u(x,t) = f(x,t), \qquad \text{in } \mathbb{R}^n \times (0,T), \tag{0.1}$$

$$u(\cdot,0) = u_0(x), \qquad \text{in } \mathbb{R}^n, \tag{0.2}$$

where $0 < T \leq \infty$, $2 \leq n \in \mathbb{N}$, $\alpha \in \mathbb{N}$ and $u(x,t)$ in (0.1), (0.2) is a scalar function. The case $\alpha = 1$ corresponds to the classical heat equation. In order to apply the results to the generalized Navier-Stokes equations we choose $f = Du^2 = \sum_{j=1}^{n} \frac{\partial}{\partial x_i} u^2$.

We assume for the initial data $u_0 \in A^\sigma_{p,q}(\mathbb{R}^n)$ with $A \in \{B, F\}$, that $\sigma > \frac{n}{p}$, $1 \leq p, q \leq \infty$, i.e. that spaces are multiplication algebras. Later we lower this assumption to $\frac{n}{p} - 2\alpha + 1 < \sigma < \frac{n}{p}$. Then these spaces cover all supercritical cases for the initial data. We show the existence and uniquness of solutions u belonging to some spaces $L_{2\alpha v}((0,T), \frac{a}{2\alpha}, A^\sigma_{p,q}(\mathbb{R}^n))$, where

$$\|u|L_{2\alpha v}((0,T), \frac{a}{2\alpha}, A^\sigma_{p,q}(\mathbb{R}^n))\| = \left(\int_0^T t^{av} \|u(\cdot,t)|A^\sigma_{p,q}(\mathbb{R}^n)\|^{2\alpha v} \mathrm{d}t \right)^{1/2\alpha v} < \infty.$$

*franka.baaske@uni-jena.de